

# Counting

<b>Multiplication rule and tree diagrams</b>		<table border="1"> <tr> <td>Symbol</td> <td>Number of types of</td> </tr> <tr> <td><math>n_{\star}</math></td> <td>stars</td> </tr> <tr> <td><math>n_{\square}</math></td> <td>boxes for each star</td> </tr> <tr> <td><math>n_{\circ}</math></td> <td>disks for each box</td> </tr> </table>	Symbol	Number of types of	$n_{\star}$	stars	$n_{\square}$	boxes for each star	$n_{\circ}$	disks for each box	<table border="1"> <tr> <td>Number of leaves</td> </tr> <tr> <td><math>= n_{\star} \cdot n_{\square} \cdot n_{\circ}</math></td> </tr> </table>	Number of leaves	$= n_{\star} \cdot n_{\square} \cdot n_{\circ}$
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<b>Factorial</b>	$n! := n \cdot (n - 1) \cdot (n - 2) \cdot \dots \cdot 2 \cdot 1$ $0! := 1$												
<b>Permutations</b>		Number of distinct ordered arrangements that can be produced by placing $r$ of $n$ distinct objects in $r$ distinct locations. ${}_n P_r = \frac{n!}{(n - r)!}$											
<b>Permutations with repetitions</b>	<p>Number of spellings (neglecting distinguishing subscripts)</p> <p><math>r_B!</math> permutations of B "among slots" available for B</p> <p><math>r_A!</math> permutations of As among slots available for As</p> <p><math>r_N!</math> permutations of Ns among slots available for Ns</p> <p><math>n!</math> permutations of <math>n</math> distinguishable letters</p>												
	Number of distinct ordered arrangements of $n$ objects, $r_1$ of which are indistinguishable copies of one item, $r_2$ of which are indistinguishable copies of another item, etc.		$\frac{n!}{r_1! r_2! \dots}$										
<b>Combinations (interpretation I: arrangements of stars and balls)</b>		Number of ways to arrange $k$ stars and $n - k$ balls in $n$ distinct locations ${}_n C_k = \frac{n!}{(n - k)! k!}$											
<b>Combinations (interpretation II: objects in hand)</b>		Number of ways to grab $r$ objects in hand from among $n$ distinct objects. ${}_n C_r = \frac{n!}{(n - r)! r!}$											
<b>Using combinations (interpretation III: stars in boxes)</b>	$k$ stars in $n$ distinguishable boxes:  $k$ stars and $n + 1$ walls: 		Number of ways to allocate $k$ indistinguishable stars among $n$ distinguishable boxes $k + (n + 1) - 2 C_k$										